APPLIED STATISTICS

Demographic methods

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CONTENTS

Keywords

Vital events; Sex ratio; Mortality; Life table; Fertility; Age pyramid; Population growth; Mortality curves

Sources of demographic data

The total number of persons inhabiting a given region constitute its population. Demography deals with the measurement of certain characteristics of the population (its major concern being the measurement of growth in the population).

Vital Statistics is a branch of Biometry that deals with the analysis of the data pertaining to vital events such as births, deaths, marriages, divorce, sickness, migration, adoption etc.

The data on vital statistics may be obtained from the following sources:

a) **Census:** All over the world almost in all countries, population censuses are conducted generally at ten-year intervals. During a census, information is collected regarding the number of persons inhabiting an area, their age, sex, marital status, occupation, religion and other economic and social characteristics.

b) **Vital Statistics Registers:** Generally in many countries the important vital events like births and deaths are registered under the law. In addition to their statistical utility these data also serve as legal documents.

c) **Hospital Records:** Every hospital and a good nursing home provides information on each patient, his age, sex, nature of illness, type of treatment administered etc.

d) **Adhoc Surveys:** Adhoc surveys are conducted in countries with faulty registration system to collect information on vital events. NSSO in our country is responsible for collecting such information.

Demographic profiles of Indian census

Census means to count the number of people residing in an area at a given point of time. In almost all countries of the world censuses are carried out, generally at ten-years interval. In India the first census was taken in 1871-72. However, it did not cover the entire country. The next census was taken in 1881 on a uniform basis over the entire country. Since then we have been having a population census regularly at 10-year intervals.

There are two methods of counting the number of people viz. de jure method and de facto method. In the de jure method a person is counted in his usual place of residence. In the de facto method a person is counted where he is actually found on the reference date of census. Till 1931, de facto method was being used. However, since 1941 we have been using the de jure method-rather a combination of the two methods.

During a census information is collected about age, sex, religion, occupation, number of inmates, their education, number of dependents, any births/deaths that would have take place in the inter censal period etc. In 1948, a census act was passed, because of which people are now under legal obligation to give information to the census authorities and authorities are also legally bound to utilize this information for statistical purposes only.

There are two methods of collecting census data:

1. The canvasser method
2. The household method

In the first method, an enumerator approaches each household allotted to him and notes down information himself after consulting any senior member of the household. In the second method the enumerator distributes the census data form to the households and the head of the family is expected to fill in the form giving required information about all members of the family. The filled forms are then collected by the enumerator after the census date for further processing.

Some of the distinct features of census of India, 2001 are as follows:

The period of enumeration was from 9th February2001 to 28th February 2001. The house listing operation was completed by May 2000. Five million enumerators were employed to carry out the census operation. The reference date was 1st March 2001 and some revisional rounds were made between 1st March and 5th March. The census schedule was published in 16 languages. Contrary to the schedule of 1991, this schedule had only the house list and household schedule, the individual slip had been done away with. Questions in the house list remained the same. Table -1 gives the Population and Sex ratio for India for a few census years.

Table 1

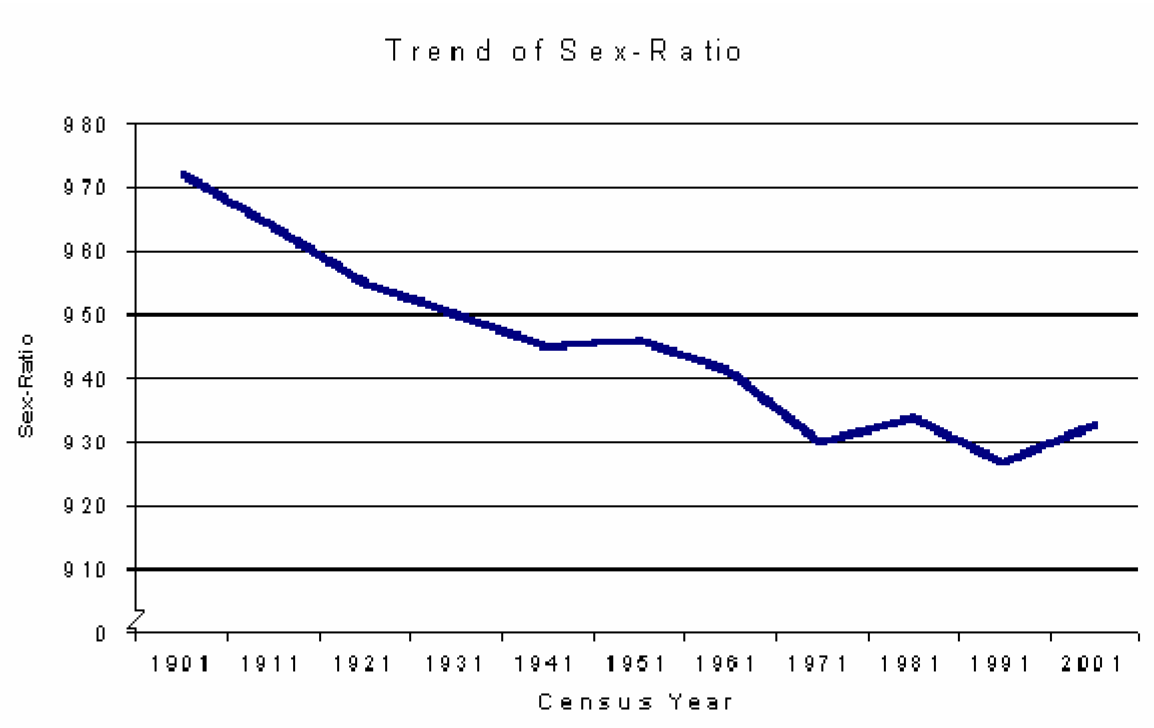
Census Year	Population (in Million)	Sex-Ratio
1901	238.4	972
1911	252.1	964
1921	251.3	955
1931	279.0	950
1941	318.7	945
1951	361.1	946
1961	439.2	941
1971	548.2	930
1981	683.3	934
1991	846.4	927
2001	1028.7	933

Table 2 gives the % distribution of population by age and sex for India for the year 2002.

Table 2

% Distribution of Population by Age Group to Total Population 2002			
Age	Total	Male	Female
0-4	11.3	11.5	11
5-9	10.3	10.5	10.2
10-14	11.5	11.6	11.3
15-59	59.8	59.7	59.9
60+	7.1	6.7	7.4

Graph -1 shows the trend of sex-ratio for India for the census years 1901 - 2001.



Graph 1

Measurement of mortality

The raw data on vital events are available in the form of frequencies of the occurrence of these events. To attach statistical significance to these data, these are generally converted to rates.

$$\text{Rate of a vital event} = \frac{\text{Number of cases of the vital event}}{\text{Total number of persons exposed to the risk of occurrence of the event}} \qquad (1)$$

As the population of a place keeps changing, thus the usual practice is to take the mid period population or the average population during the period of study in the denominator.

Crude death rate

It gives the number of deaths that occur on the average per 1000 people in the given region. It is the simplest of all the measures of mortality and may be defined as:

$$\text{CDR} = \text{M} = \frac{\text{Total number of deaths from all causes in the given region in the given time period}}{\text{Average size of the population of the given region in the given time period}} \times k$$
$$= \frac{D}{P} \times k \qquad (2)$$

where, k =1000, usually.

Crude death rate as defined above is a probability rate. It gives the probability that a person belonging to the given population will die in the given period.

However, Crude death rate completely ignores the age and sex distribution of the population and the other factors such as race, occupation, geographical area that cause differential mortality. Thus it is not suitable for comparing the mortality conditions prevailing in two regions altogether with different age and sex distributions or of same region in two different time periods. In case, the age and sex distributions of the two places under comparison do not show much variation or the two time periods under comparison are not far from each other, CDR may be the right choice because of its ease in calculations.

Countries, in which the proportion of old people is higher, will exhibit a high value of CDR. Also countries in which the average longevity per person is low, will show a higher value of CDR. Table -3 gives the value of crude death rate for males and females for India for rural and urban population for the years 2002 and 2004.

Table 3

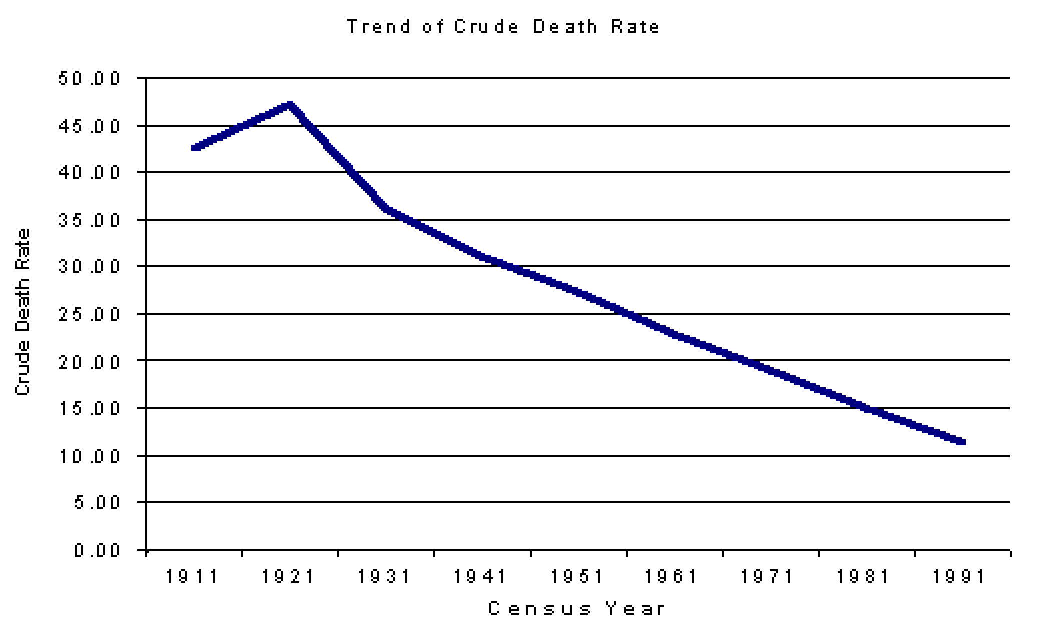
Crude Death Rate of India						
	(2002)			(2004)		
	TOTAL	RURAL	URBAN	TOTAL	RURAL	URBAN
MALE	8.4	9	6.5	8.0	8.7	6.1
FEMALE	7.7	8.4	5.6	7.0	7.6	5.4
TOTAL	8.1	8.7	6.1	7.5	8.2	5.8

Table 4 gives the value of CDR for India for census years 1911 - 1991.

Table 4

Census Year	Crude Death Rate
1911	42.6
1921	47.2
1931	36.3
1941	31.2
1951	27.4
1961	22.8
1971	19.0
1981	15.0
1991	11.4

Graph 2 shows the trend of crude death rate for India for the census period 1911 - 1991.



Graph –2

Specific death rate

It is an improvement over crude death rate as it takes into account the age and sex composition of the population. It also provides an important component for constructing net reproduction rates and life tables. The death rate which is computed for a specific segment of the population may be termed as SDR. It is given as:

$$\text{SDR}=\frac{\text{Total number of deaths in the specified section of the population during the given time period}}{\text{Total population in the specified section of the population during the given time period}}\times k \qquad (3)$$

where, k=1000, usually.

SDRs are generally computed specific to age or sex or both. However, there are many other factors such as occupation, race, marital status, topographical factors causing variations in the mortality, which should also be taken into account.

Specific death rate is not suitable for comparing the overall mortality conditions prevailing in two regions as mortality may be higher in certain age groups in one region while lower in the other age groups.

Age specific death rate

Let ${}_nD_x$ be the number of deaths of persons aged between x and (x+n) in a given community during a given period and ${}_nP_x$ be the number of persons in the same age group in the given community during the given period, then the age specific death rate for the age group x to (x+n) is:

$${}_nM_x=\frac{{}_nD_x}{{}_nP_x}\times k \qquad (4)$$

Let ${}^{m}_{n}P_x$ and ${}^{m}_{n}D_x$ represent the number of males in the age group x to (x+n) and the number of deaths occurring to such males, then SDR for males in the age group x to (x+n) is given as:

$${}^{m}_{n}M_x = \frac{{}^{m}_{n}D_x}{{}^{m}_{n}P_x} \times k \qquad (5)$$

The above death rate is specific to both age and sex.
Further if n = 1, the annual age specific death rate is

$$M_x = \frac{D_x}{P_x} \times k \qquad (6)$$

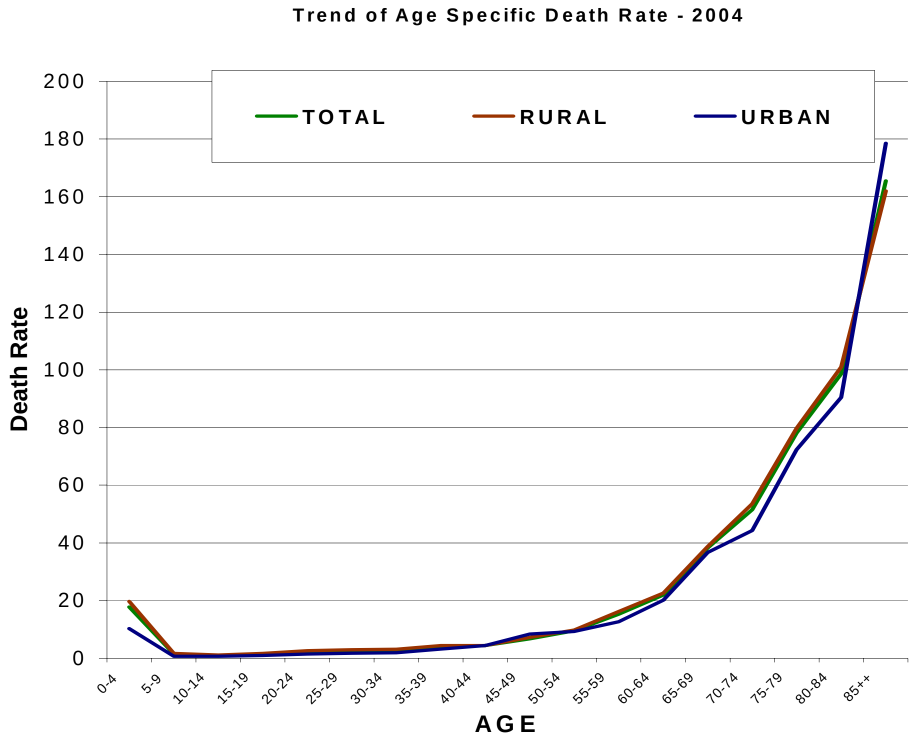
where, k =1000, usually.

Table -5 gives the specific death rate for males and females for rural and urban population for India for the year 2004

Table 5

Age Specific Death Rate By Sex – 2004									
AGE	TOTAL			RURAL			URBAN		
	TOTAL	MALE	FEMALE	TOTAL	MALE	FEMALE	TOTAL	MALE	FEMALE
0-4	17.8	17.0	8.6	19.7	18.8	20.7	10.3	10.3	10.2
5-9	1.5	1.4	1.6	1.7	1.6	1.8	0.7	0.6	0.8
10-14	1.0	1.0	1.1	1.1	1.0	1.1	0.7	0.7	0.8
15-19	1.5	1.4	1.7	1.7	1.5	2.0	1.0	1.1	1.0
20-24	2.3	2.1	2.5	2.6	2.4	2.9	1.5	1.4	1.6
25-29	2.6	2.8	2.5	3.0	3.2	2.8	1.8	1.8	1.7
30-34	2.8	3.3	2.3	3.1	3.5	2.7	1.9	2.6	1.2
35-39	4.1	4.7	3.4	4.4	4.8	4.0	3.2	4.4	1.9
40-44	4.4	5.3	3.4	4.4	5.2	3.5	4.4	5.7	2.9
45-49	6.7	8.0	5.3	7.2	8.4	6.0	8.4	10.6	6.4
50-54	9.5	11.7	7.1	9.8	12.1	7.4	9.3	12.8	8.5
55-59	15.3	18.1	12.4	16.2	19.1	13.3	12.7	15.4	9.8
60-64	22.0	25.4	18.7	22.6	26.2	19.1	20.2	22.9	17.4
65-69	38.3	43.5	33.5	38.8	44.9	33.2	36.7	38.8	34.7
70-74	51.5	55.7	47.7	53.6	58.7	49.0	44.3	45.5	43.3
75-79	78.1	83.7	73.1	79.7	86.9	73.1	72.3	71.6	72.9
80-84	98.6	105.3	92.6	101.0	107.2	95.6	90.4	99.0	83.4
85++	165.5	175.3	157.2	162.0	169.3	155.3	178.5	195.8	164.0

Graph 3 shows the trend of age specific death rate for rural, urban and total population for India for the year 2004.



Graph 3

Standard death rate (STDR)

In order to compare the mortality conditions prevailing in two different regions (say A and B) or of the same region over two different time periods, one has to take into account a number of factors such as age, sex, occupation, race, marital status etc. Generally, it is difficult to gather such a large amount of information which may not be of much use otherwise. Thus to reach at a single index of mortality based on some kind of average of the death rates for various segments of the population, the simplest measure that one may use is CDR. However, the value of CDR is greatly affected by the age and sex distribution of the population under consideration. For comparing the mortality situation of two regions with altogether different age and sex distributions, the results obtained by CDR will not be valid. Consider the CDR of two regions A and B in terms of the age specific death rates:-

$$CDR^A = M^A = \frac{D^A}{P^A} \times 1000 = \frac{\sum_x M_x^A P_x^A}{\sum_x P_x^A}$$

$$CDR^B = M^B = \frac{D^B}{P^B} \times 1000 = \frac{\sum_x M_x^B P_x^B}{\sum_x P_x^B} \qquad (7)$$

Expression (7) shows CDR as the weighted arithmetic mean of the age specific death rates, weights being equal to the proportion of population with corresponding ages i.e. $\frac{P_x}{\sum_x P_x}$.

Now even if $M_x^A = M_x^B \ \forall x$,

$M^A \neq M^B$ because of the differences in population proportions $\frac{P_x^A}{\sum_x P_x^A}$ and $\frac{P_x^B}{\sum_x P_x^B}$ i.e. even if the age specific death rates in the two populations are comparable, their CDRs will differ as a consequence of the difference in the age distributions of the two populations.

To remove this defect we have to use the same set of weights in taking the weighted average of the SDRs of two regions. The index thus obtained is called standardized death rate or adjusted death rate. When age distribution is taken as the basis of finding the standard set of weights, the standardized death rate will be called age standardized death rate or age adjusted death rate. Similarly, it may be adjusted for other characteristics such as sex, occupation etc. and similarly interpreted. A death rate may also be adjusted for more than one factor simultaneously. There are two methods of standardization:

1. Direct Method of Standardization: In this method we take some other population called a standard population. For computing age adjusted death rate we take the population for different age groups in the standard population as the set of weights.

Let P_x^S be the number of persons at age x in the standard population, then standardized death rate or adjusted death rate for the two regions A and B are given as:

$$STDR^A = \frac{\sum_x M_x^A P_x^S}{\sum_x P_x^S}; \quad STDR^B = \frac{\sum_x M_x^B P_x^S}{\sum_x P_x^S} \tag{8}$$

Thus the age-adjusted death rate is simply CDR that would be observed in the standard population if it were subject to the age SDR of the given region.

The standard population is generally taken to be the actual population of a bigger region of which both A and B are subsets. For example, if the two regions under comparison are Punjab and Haryana, one may take the population of Northern India or whole of India as standard. The following expression shows the simultaneous standardization of age and sex:

$$\frac{\left[\sum_x {}^m M_x^A \cdot {}^m P_x^S + \sum_x {}^f M_x^A \cdot {}^f P_x^S\right]}{\left[\sum_x {}^m P_x^S + \sum_x {}^f P_x^S\right]} \tag{9}$$

Thus standardized death rate is a good index for comparing the mortality conditions of two regions. Any differences in their age specific death rates will be faithfully reflected.

2 Indirect Method of Standardization: Expression (8) requires the knowledge of the number of persons and the SDRs for various age segments in the given region. Quite often we have the classification of population by age but the SDRs for various age groups may not be available. However, the total number of deaths in the given region may be known to us. In such a case we use indirect method of standardization. This formula requires the knowledge of the number of persons and SDRs for various age groups in the standard population along with the values of P_x^A for the given region. Thus the formula for indirect age adjusted death rate is given by:

$$\text{STDR} = \frac{\text{Total number of deaths in region A}}{\sum_x P_x^A} \times \frac{\sum_x M_x^s . P_x^s \Big/ \sum_x P_x^s}{\sum_x M_x^s . P_x^A \Big/ \sum_x P_x^A}$$
$$= \text{CDR}_A \times \frac{\sum_x M_x^s . P_x^s \Big/ \sum_x P_x^s}{\sum_x M_x^s . P_x^A \Big/ \sum_x P_x^A} \qquad (10)$$

Standardized death rate suffers from the drawback that the value of STDR so obtained depends on the age and sex composition of the standard population chosen which may be quite different from that of the region under consideration. In case there is not much variation in the age distribution of the standard population from that of the region under consideration, there is not much problem. However, if the age distributions of the two regions under comparison differ significantly, it is better to take the population of that region as standard whose death rate is of more concern to us. Also choice of the standard region is quite subjective and may introduce bias in the results.

Indirect method of standardization is used whenever the requisite data for computing STDR by Direct method is not available. However, the two methods would give exactly equivalent results if the SDRs of the given region happen to be proportional to the SDRs of the standard region.

Death rate by cause

This rate measures the contribution made by a specific cause of death say, a specific disease or accident to the overall motility of a region. The death rate by cause for cause 'i' is given as:

$$M^i = \frac{D^i}{P} \times 100{,}000 \qquad (11)$$

where,

D^i = is the total number of deaths from cause 'i' in the given region in the given time period;

P = the total population of the given community in the given period;

In order to avoid fractions, the above rate has been multiplied by 100,000.

This rate serves as the basis of many public health programmes. However, like CDR it does not take into account the age and sex distribution of the population. Also it cannot be given the probability interpretation because of the way it has been defined.

Infant mortality rate (IMR)

It may be defined as:

$$\text{IMR} = \frac{\text{Number of deaths between birth and age one}}{\text{Number of live births}} \times k = \frac{D_o}{B} \times k \qquad (12)$$

where, k=1000, usually. In the computation of IMR no still birth and fetal deaths are included.

Infant mortality rate is used in lieu of ASDR for age 0 *l*.b.d. (last birthday), as the latter tends to be highly over stated (because of under enumeration of infants), where

$$\text{ASDR (for age 0 } l\text{.b.d)} = \frac{\text{Number of live births}}{\text{Number of infants}}$$

Infant mortality rate can be calculated for any population and for any time as it does not depend on data of population censuses. It can be computed specific to sex, cause of death etc. Infant mortality rate serves as an excellent indicator of the general wellbeing of the community as infant deaths are highly responsive to improvements in medical conditions. For the past many years its value has been on decline indicating an improvement in the health facilities to the citizens. However, it is not a probability rate because of the way it has been defined. Many of the deaths that occur this year may be of infants who were born last year. Similarly some of the infants who are born this year may die next year. Thus numerator and denominator in expression (12) are not exactly related. Also its value tends to be generally larger than what it should be, because of under registration of live births. Further, the definition of a live birth and still birth varies from one place to another. Table 6 gives the IMR values for India from 1971 – 2003

Table 6

Infant Mortality Rate For India			
Year	**Rural**	**Urban**	**Combined**
1971	138	82	129
1972	150	85	139
1973	143	89	134
1974	136	74	126
1975	151	84	140
1976	139	80	129
1977	140	81	130
1978	137	74	127
1979	130	72	120

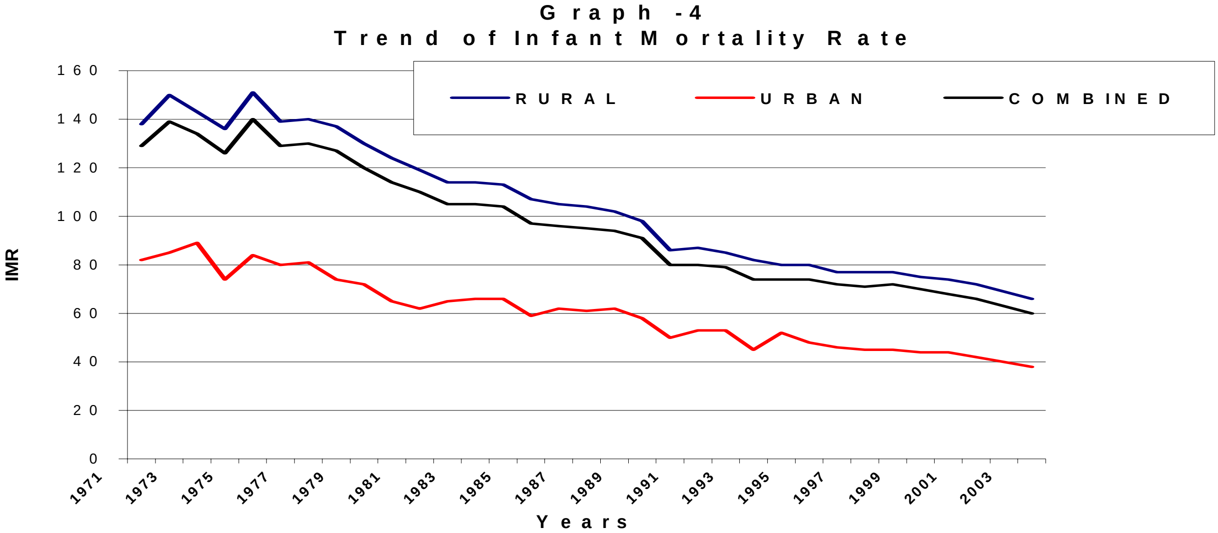
Infant Mortality Rate For India			
Year	**Rural**	**Urban**	**Combined**
1980	124	65	114
1981	119	62	110
1982	114	65	105
1983	114	66	105
1984	113	66	104
1985	107	59	97
1986	105	62	96
1987	104	61	95
1988	102	62	94
1989	98	58	91
1990	86	50	80
1991	87	53	80
1992	85	53	79
1993	82	45	74
1994	80	52	74
1995	80	48	74
1996	77	46	72
1997	77	45	71
1998	77	45	72
1999	75	44	70
2000	74	44	68
2001	72	42	66
2002	69	40	63
2003	66	38	60

Table 7 gives the Infant mortality rate by sex for rural and urban population for India for the year 2004

Table 7

Infant Mortality Rate For The Year 2004			
	Total	Rural	Urban
Male	58	64	39
Female	58	63	40
Total	58	64	40

The graphical representation of the data given in Table 6 is as follows:



EXAMPLE – 1

The following table gives the age specific death rates for males and females for two states A and B along with the age-sex distribution of a standard population. Compute STDRs for both the states and compare.

AGE	Specific Death Rates for State - A		Specific Death Rates for State - B		Standard Population	
	Male	Female	Male	Female	Male	Female
	${}^{m}M_x^A$	${}^{f}M_x^A$	${}^{m}M_x^B$	${}^{f}M_x^B$	${}^{m}P_x^S$	${}^{f}P_x^S$
0-4	18.80	20.70	68.60	72.50	4288	5349
5-9	1.60	1.80	4.10	5.30	3801	4526
10-14	1.00	1.10	17.00	8.60	3704	3524
15-19	1.50	2.00	1.40	1.60	3617	3215
20-24	2.40	2.90	1.00	1.10	3518	3154
25-29	3.20	2.80	1.40	1.70	3407	3098
30-34	3.50	2.70	2.10	2.50	3221	3012
35-39	4.80	4.00	2.80	2.50	2945	2984
40-44	5.20	3.50	3.30	2.30	2633	2875
45-49	8.40	6.00	4.70	3.40	2323	2458
50-54	12.10	7.40	5.30	3.40	2006	1987
55-59	19.10	13.30	8.00	5.30	1678	1547
60-64	26.20	19.10	11.70	7.10	1342	1298
65-69	44.90	33.20	18.10	12.40	1015	1047
70-74	58.70	49.00	25.40	18.70	717	859
75-79	86.90	73.10	43.50	33.50	654	735
80-84	107.20	95.60	55.70	47.70	631	671
85++	169.30	155.30	83.70	73.10	584	598
Total					42084	42937

SOLUTION

For the computation of STDR consider the following table:

AGE	${}^{m}M_x^A . {}^{m}P_x^S$	${}^{f}M_x^A . {}^{f}P_x^S$	${}^{m}M_x^B . {}^{m}P_x^S$	${}^{f}M_x^B . {}^{f}P_x^S$
0-4	80614.4	110724.3	294156.8	387802.5
5-9	6081.6	8146.8	15584.1	23987.8
10-14	3704.0	3876.4	62968.0	30306.4
15-19	5425.5	6430.0	5063.8	5144.0
20-24	8443.2	9146.6	3518.0	3469.4
25-29	10902.4	8674.4	4769.8	5266.6
30-34	11273.5	8132.4	6764.1	7530.0
35-39	14136.0	11936.0	8246.0	7460.0
40-44	13691.6	10062.5	8688.9	6612.5
45-49	19513.2	14748.0	10918.1	8357.2
50-54	24272.6	14703.8	10631.8	6755.8
55-59	32049.8	20575.1	13424.0	8199.1
60-64	35160.4	24791.8	15701.4	9215.8
65-69	45573.5	34760.4	18371.5	12982.8
70-74	42087.9	42091.0	18211.8	16063.3
75-79	56832.6	53728.5	28449.0	24622.5
80-84	67643.2	64147.6	35146.7	32006.7
85++	98871.2	92869.4	48880.8	43713.8
Total	576276.6	539545.0	609494.6	639496.2

$STDR^A =$ 13.12407 (per thousand) $STDR^B =$ 14.69038 (per thousand)

The STDRs of the two states show that the average number of deaths in the two states is approximately the same.

Complete life table

A life table gives the mortality experience of a hypothetical group of people (called the cohort) starting life together and experiencing same mortality conditions as given by the observed age specific death rate throughout their lifetime. It shows how this cohort gets depleted through deaths at each age till finally nobody is alive. One can compute the probability that a person of some given age will live for a specific number of years. It also enables to compute the average longevity per person. The data for the construction of life tables is taken from census and death registers. As a number of factors cause differential mortality, life tables may be constructed on the basis of each one of them such as religion, sex, occupation etc.

Basic assumptions in the construction of a life table

1) Start with a hypothetical group of infants say 1,00,000 all born at the same time.

2) The deaths are distributed uniformly over the age interval (x, x+1) (an assumption which is not valid for early years of life, especially for age 0).
3) There is no change in the cohort but for deaths i.e. it is closed to migration.
4) At all ages individuals die according to a predetermined fixed schedule.

Description of various columns of a life table

A life table consists of the following columns:

i)	x	Age on the last birthday where x takes non-negative integral values.
ii)	l_x	Number of persons living at any specified age x.
iii)	l_0	Cohort or radix of life table.
iv)	d_x	Number of persons who attain age x and die before reaching the age $(x+1) = l_x - l_{x+1}$
v)	q_x	The probability that a person of exact age x will die before reaching the age (x+1) $= d_x / l_x$
vi)	p_x	The probability that a person of exact age x will survive till the age (x+1) $= l_{x+1} / l_x$
vii)	m_x	Probability that a person in the age group x to (x+1) will die while in this age group. $= 2q_x / (2-q_x)$
viii)	μ_x	Ratio of instantaneous rate of decrease in l_x to the value of l_x; $= -\frac{d\log l_x}{dx}$
ix)	L_x	Number of years lived in the aggregate by the cohort of l_0 persons between ages x and (x+1); $= \frac{1}{2}(l_x + l_{x+1})$
x)	T_x	Total number of years lived by the cohort after attaining age x $= L_x + L_{x+1} .....$
xi)	e_x^o	Complete expectation of life at age x is the average number of years lived after age x. $= T_x / l_x$
	e_0^0	Expectation of life at birth.
xii)	e_x	Average number of complete years lived by the cohort after age x. $= e_x^0 - \frac{1}{2}$

NOTE: q_x column is called the pivotal column of a life table. Once the values of l_0 and q_x are known, the entire life table can be constructed. The values of q_x can be obtained from the relationship $q_x = \frac{2m_x}{2+m_x}$, where the values of m_x are calculated from the relationship $m_x = \frac{d_x}{L_x}$ on the basis of census data and death registration data.

The following table gives the expectation of life for male and female at birth for India:

Table 8

Expectation of Life at Birth for the Period 1970 - 2002			
Period	Males	Females	Combined
1970-1975	50.5	49.0	49.7
1976-1980	52.5	52.1	52.3
1981-1985	55.4	55.7	55.4
1986-1990	57.7	58.1	57.7
1993-1997	60.4	61.8	61.1
1995-1999	60.8	62.5	61.7
1997-2001	61.3	63.0	62.2
1998-2002	61.6	63.3	62.5

EXAMPLE - 2

The following table gives a part of the census data of female population in India. Using this information compute the other columns of the life table.

X	0	1	2	3	4	5	6	7	8	9	10
q_x	0.1383	0.0362	0.0286	0.0223	0.017	0.0128	0.0094	0.007	0.0052	0.0041	0.0035

(Given $T_{10} = 3266067$)

SOLUTION

Life Table for Females of India based on census data

X	l_x	d_x	q_x	p_x	m_x	L_x	T_x	e_x^o
0	100000	13826	0.1383	0.8617	0.1485	93087	4062260	40.6226
1	86174	3119	0.0362	0.9638	0.0369	84615	3969173	46.0600
2	83055	2378	0.0286	0.9714	0.0290	81866	3884559	46.7709
3	80677	1797	0.0223	0.9777	0.0225	79779	3802693	47.1348
4	78880	1343	0.0170	0.9830	0.0172	78209	3722914	47.1972
5	77537	991	0.0128	0.9872	0.0129	77042	3644706	47.0060
6	76546	723	0.0094	0.9906	0.0095	76185	3567664	46.6081
7	75823	527	0.0070	0.9930	0.0070	75560	3491480	46.0478
8	75296	391	0.0052	0.9948	0.0052	75101	3415920	45.3666
9	74905	305	0.0041	0.9959	0.0041	74753	3340820	44.6008
10	74600	261	0.0035	0.9965	0.0035	74470	3266067	43.7811

Uses of life tables

Life tables are extensively used by demographers, public health programmers, socio-economic decision-makers, insurance agencies, actuaries, Government etc. The fields of life insurance and actuarial science revolve around life tables. These tables were originally devised to meet the requirements of insurance agencies. They enable in deciding the amount of premium payable under different policies for various age groups. They help in the computation of net reproduction rate so as to study population growth. They are used by the government for determining retirement benefits for its employees, for knowing the number of senior citizens, for knowing the future size of the population for estimating the school going population etc. In case information on cause of death is also available then this can be incorporated in a life-table to find the probability of death due to a specific cause. Further, one could also study the impact of this cause on expectation of life.

Measurement of fertility

Another important vital event is births which are responsible for increasing the size of the population. A closely related concept is that of fertility. Fertility refers to the actual production of children. Fertility should not be confused with fecundity which means capacity to produce children. Only a particular section of the females has the capacity to bear children viz. females belonging to the age group 15 to 49. Further, it is to be noted that only live births are to be taken into account while measuring fertility as it is only a live birth which accounts for increase in population.

Crude birth rate (CBR)

It is defined as:

$$\text{CBR}=\frac{\text{Total number of live births in the given region in the given time period t}}{\text{Total size of the population in the given region in the given time t}}\times k$$

$$=\frac{B^t}{P^t}\times 1000 \quad \text{....(13)}$$

where, k=1000, usually

It is the simplest of all the measures of fertility. However, it completely ignores the age and sex distribution of the population. Also it is not a probability rate as the whole population is not exposed to the risk of bearing children. Crude birth rate is unsuitable for comparison purposes as it assumes that women in all the ages have the same fertility. It can not be used to compare the fertility conditions prevailing in different countries, as childbearing age groups are not identical in all the countries. In countries with warmer climatic conditions females approach the reproductive period much earlier than in countries with colder climate.

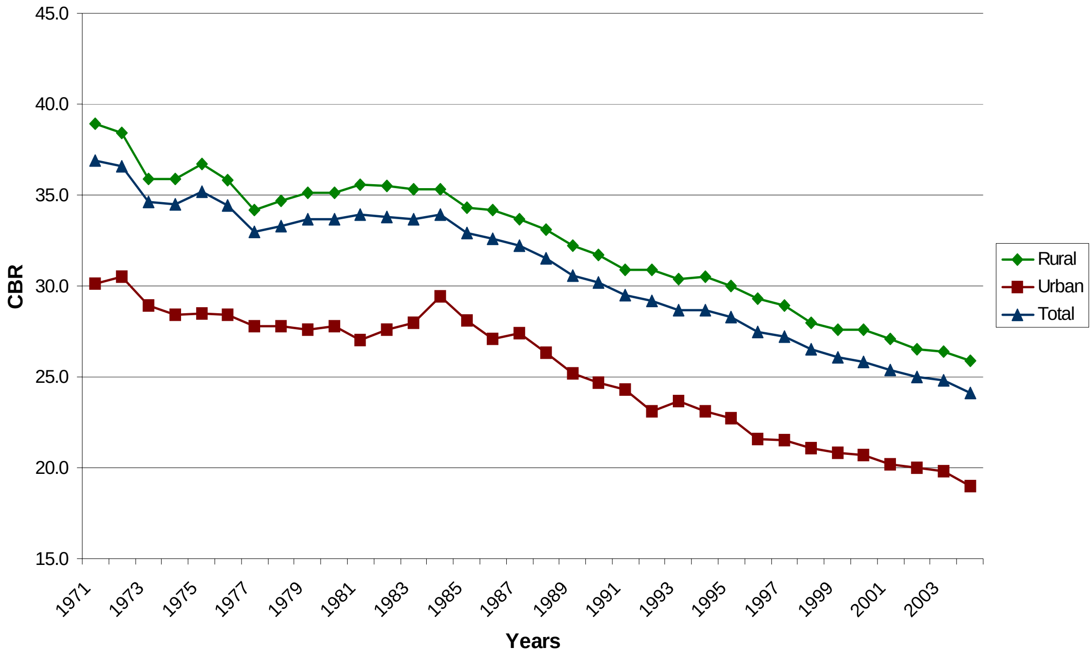
Also countries, which have a higher number of females in the reproductive ages or have a higher rate of fertility will exhibit a higher value of CBR. The value of CBR also gets affected by other factors such as sex ratio, family planning measures, marriage rate, etc. -9 gives the CBR for India from 1971-2004.

Table 9

Crude Birth Rates For India From 1971 - 2004			
Year	**Rural**	**Urban**	**Total**
1971	38.9	30.1	36.9
1972	38.4	30.5	36.6
1973	35.9	28.9	34.6
1974	35.9	28.4	34.5
1975	36.7	28.5	35.2
1976	35.8	28.4	34.4
1977	34.2	27.8	33.0
1978	34.7	27.8	33.3
1979	35.1	27.6	33.7
1980	35.1	27.8	33.7
1981	35.6	27.0	33.9
1982	35.5	27.6	33.8
1983	35.3	28.0	33.7
1984	35.3	29.4	33.9
1985	34.3	28.1	32.9
1986	34.2	27.1	32.6
1987	33.7	27.4	32.2
1988	33.1	26.3	31.5
1989	32.2	25.2	30.6
1990	31.7	24.7	30.2
1991	30.9	24.3	29.5
1992	30.9	23.1	29.2
1993	30.4	23.7	28.7
1994	30.5	23.1	28.7
1995	30.0	22.7	28.3
1996	29.3	21.6	27.5
1997	28.9	21.5	27.2
1998	28.0	21.1	26.5
1999	27.6	20.8	26.1
2000	27.6	20.7	25.8
2001	27.1	20.2	25.4
2002	26.5	20.0	25.0
2003	26.4	19.8	24.8
2004	25.9	19.0	24.1

The corresponding graphical representation of the data is as follows:

GRAPH - 5
Trend of Crude Birth Rate



General fertility rate (GFR)

It relates the number of live births to the number of females in the child-bearing ages. It is defined as:

$$GFR = \frac{\text{Number of live births in the given region in the given time period t}}{\text{Number of females in the child bearing age in the given region in the given time period t}} \times k$$

$$= \frac{B^t}{\sum_{\lambda_1}^{\lambda_2} {}^fP^t} \qquad (14)$$

where k=1000, usually and λ_1, λ_2 are the lower and upper limits of the female child bearing age that are generally taken as 15 and 49 respectively. Births occurring outside this range are included in the age group 15 and 49 respectively.

General fertility rate is a probability rate. Also, it takes into account the sex composition of the population and age composition to some extent only. It assumes that females belonging to different age groups are exposed to the uniform risk of bearing children. Thus it is quite possible that two populations with altogether different general fertility rate may have the same fertility in each one year age group or two population with approximately the same value of GFR may show entirely different status of fertility. As such it is not suitable for comparison purposes.

The GFR for India for the year 2002 is as follows:

Total	97.1
Rural	106
Urban	72.5

Age specific fertility rate (ASFR)

In order to form a better idea of the fertility situation, it is advisable to relate the number of live births to the number of females belonging to a particular age group. Thus specific fertility rate (SFR) for the age group x to (x+n) is given as:

$$ {}_{n}i_{x} = \frac{\text{Number of live births to females in the age group x to (x+n) in the given region in the given time period}}{\text{Number of females in the age group x to (x+n) in the given region in the given time period}} \times k = \frac{{}_{n}B_{x}}{{}_{n}^{f}P_{x}} \times k \qquad (15)$$

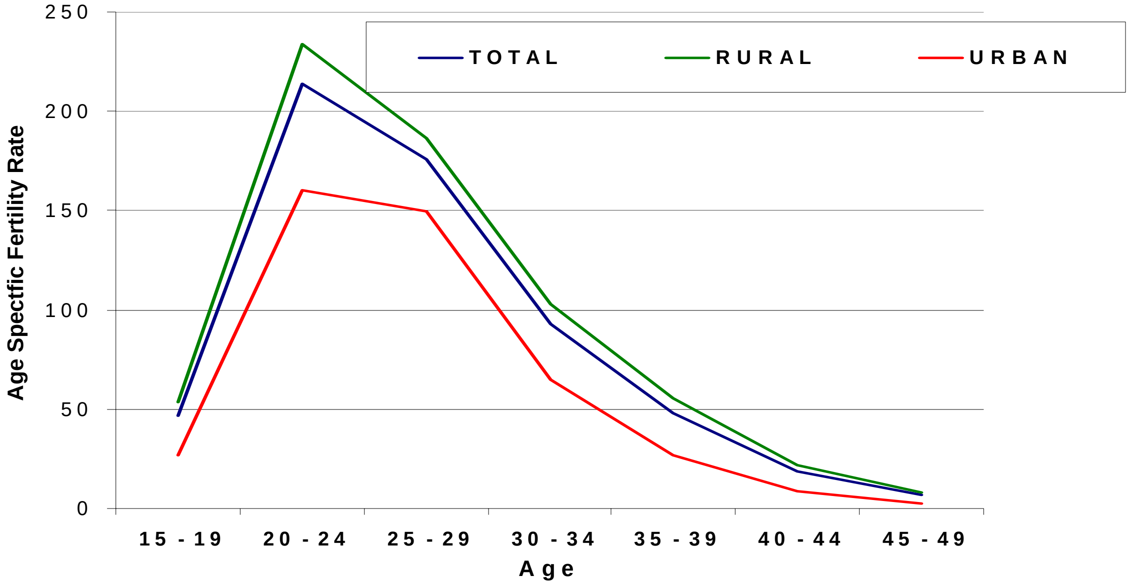
ASFR is a probability rate. It takes into account the age composition of the female population in the child-bearing ages. ASFR can also be calculated for any particular race, religion etc. However, it does not enable us to compare the over all fertility situation prevailing in two places or of the same place at two different time periods. ASFR is influenced by a number of factors such as age of a female at marriage, family planning measures adopted, divorces, separation widowhood etc. The curve obtained on plotting ${}_{n}i_{x}$ values against x is called the fertility curve. It is a positively skewed curve as specific fertility starts from a low point, rises to a peak somewhere between 20 to 29 years of age and then declines steadily. The following table gives the age specific fertility rate for India for the year 2002.

Table 10

Age Specific Fertility Rate for India -2002			
Age	**Total**	**Rural**	**Urban**
15 - 19	47.0	53.6	26.8
20 - 24	214.0	233.6	160.4
25 - 29	175.9	186.2	149.4
30 - 34	92.8	102.7	65.0
35 - 39	47.8	55.5	26.6
40 - 44	18.5	22.1	8.9
45 - 49	6.6	8.1	2.7

The following graph shows the trend of age specific fertility rate for total, rural and urban population for India for the year 2002.

GRAPH 6
Trend of Age Specific Fertility Rate



Total fertility rate (TFR)

ASFRs give a clear picture of the fertility situation prevailing in different segments of a region. However little can be said about the overall fertility situation prevailing in that region as fertility will be higher for certain age groups while lower for others. Thus in order to obtain an index of the overall fertility situation prevailing in a region the various age specific fertility rates need to be combined to reach at a standardized fertility rate called the total fertility rate. It is given as

$$\text{TFR} = \sum_{\lambda_1}^{\lambda_2} i_x = \sum_{\lambda_1}^{\lambda_2} \frac{B_x}{{}^{f}P_x} \times k \qquad (16)$$

For grouped data the value of $\text{TFR} = \sum_x n({}_n i_x)$

It is a hypothetical figure. It gives the number of babies that would be born to a cohort of 1000 females, starting life together assuming that:

1. all of them live up to age λ_2
2. at each age of the child bearing age span, all of them experience the fertility conditions given by the observed age specific fertility rate.

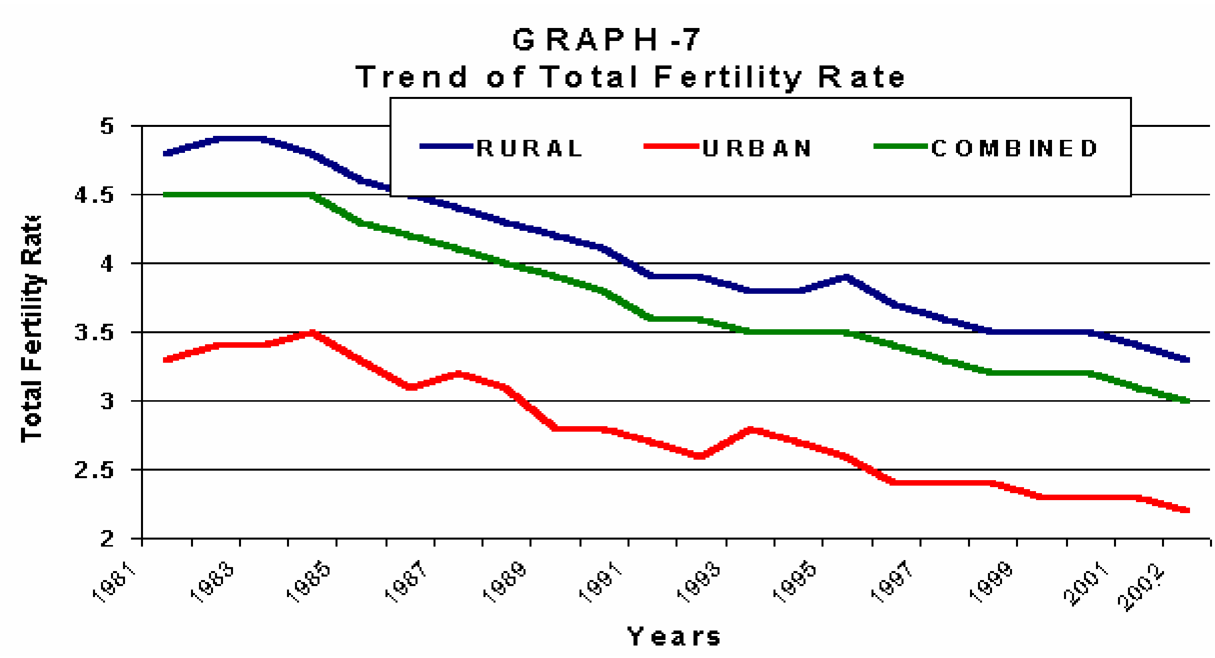
It is quite useful for comparing the fertility situations of two regions or of the same place at two different time periods.

Table 11 gives the total fertility rate for India for the period 1981 - 2002.

Table 11

Total Fertility Rate for India				Total Fertility Rate for India			
Year	**Rural**	**Urban**	**Combined**	**Year**	**Rural**	**Urban**	**Combined**
1981	4.8	3.3	4.5	1992	3.9	2.6	3.6
1982	4.9	3.4	4.5	1993	3.8	2.8	3.5
1983	4.9	3.4	4.5	1994	3.8	2.7	3.5
1984	4.8	3.5	4.5	1995	3.9	2.6	3.5
1985	4.6	3.3	4.3	1996	3.7	2.4	3.4
1986	4.5	3.1	4.2	1997	3.6	2.4	3.3
1987	4.4	3.2	4.1	1998	3.5	2.4	3.2
1988	4.3	3.1	4.0	1999	3.5	2.3	3.2
1989	4.2	2.8	3.9	2000	3.5	2.3	3.2
1990	4.1	2.8	3.8	2001	3.4	2.3	3.1
1991	3.9	2.7	3.6	2002	3.3	2.2	3.0

The following graph shows the trend of the total fertility rate for rural, urban and combined population for India for the period 1981 – 2002.



Measurement of population growth

Gross production rate (GPR)

Gross reproduction rate is a measure of the population growth. Along with considering the age and sex composition of the population it also considers the sex of the newly born baby as it is a female birth, which is ultimately going to be responsible for growth in population. Gross reproduction rate is given as:

$$GRR = \sum_{\lambda_1}^{\lambda_2} {}^{f}i_x = \sum_{\lambda_1}^{\lambda_2} \frac{{}^{f}B_x}{{}^{f}P_x} \times k \qquad (17)$$

For grouped data the value of GRR is:

$$GRR = n\sum_{\lambda_1}^{\lambda_2} {}_{n}^{f}i_x = n\sum_{\lambda_1}^{\lambda_2} \frac{{}_{n}^{f}B_x}{{}_{n}^{f}P_x} \times k \qquad (18)$$

where ${}_{n}^{f}i_x$ is the age specific fertility rate for the age group x to x+n based on female births. Thus Gross Reproduction Rate may be defined as the rate of replenishment of the female population. It is a hypothetical figure. It gives the number of females that would be born, on the average, to each of a group of women, beginning life together and assuming that

1. None of the dies before reaching the age λ_2
2. All of them experience, throughout their reproductive span the current level of fertility given by ${}^{f}i_x$

The expression (18) requires the knowledge of:

1. age of mother at the time of birth
2. sex of the newly born baby

However this information may not always be available but the total number of births be known. In such a case an approximate value of GRR may be computed from the following expression which is based on the assumption that sex ratio at birth is more or less constant over all ages of mothers:

$$GRR = TFR \times \frac{\text{Number of female births}}{\text{Total number of births}} \qquad (19)$$

Gross reproduction rate may be used for comparing the fertility situation in different regions. However, because of under registration of births and wrong statement about the age of mother at the time of registration, the value of gross reproduction rate may not be accurate. Also it assumes that none of the newly born babies dies before the age λ_2, which inflates the number of future mothers.

The GPR for India for the year 2002 is as follows:

Total	1.4
Rural	1.6
Urban	1.0

Net production rate (NPR)

NRR is another measure of population growth. Growth in population depends on the balance between births and deaths. Thus net reproduction rate is simply gross reproduction rate adjusted for the effects of mortality. It is also a hypothetical figure giving the number of female babies that would be born to a group of females beginning life together if they were subject to current rates of fertility and mortality throughout their life span. It may be defined as: -

$$NRR=k\sum_{\lambda_1}^{\lambda_2} n\frac{{}_n^fB_x}{{}_n^fP_x}\times\left(\frac{{}_n^fL_x}{{}^fl_o}\right) \qquad (20)$$

The factor $\frac{{}_n^fL_x}{{}^fl_o}$ measures the life table probability of survival of a female to the age group x to x+n. As probability is always $\leq$ 1 therefore net reproduction rate $\leq$ gross reproduction rate i.e. gross reproduction rate serves as an upper limit to net reproduction rate. If NRR =1, it implies that a population of newly born girls will exactly replace itself in the next generation provided the current fertility and mortality rates prevail in future also. In this case population will have a tendency to remain constant in size. If net reproduction rate is greater than one, population has a tendency to increase as in this case each female will be replaced by more than one daughter. Similarly if NRR is less than 1, population will tend to decline. However, net reproduction rate should not be used for forecasting the size of future population as it assumes that current rates of fertility and motility prevail in future also, an assumption, which is not true. It also ignores the factor of migration and considers the age and sex distribution of a hypothetical life table population that may be quite different from that of the actual population.

EXAMPLE - 3

Compute the values of GFR, TFR and GRR for the following data.

Age	15-19	20-24	25-29	30-34	35-39	40-44	45-49	Total
Number of Women	12000	12300	11850	11400	11100	11250	10875	80775
Number of Birth	221	1907	1610	1122	687	210	109	5866

Assume that the proportion of female births is 38.9 %

SOLUTION:

Age	Number of Women	Number of Birth	Age Specific Fertility Rate
15-19	12000	221	18
20-24	12300	1907	155
25-29	11850	1610	136
30-34	11400	1122	98
35-39	11100	687	62
40-44	11250	210	19
45-49	10875	109	10
Total	80775	5866	498

GFR = (5866/80775) x 1000 = 72.6221 per thousand
TFR = 5 x 498 = 2492 per thousand
GRR = (38.9/100) x 2492 = 969 per thousand

Age pyramid of sex composition

It is a graphical representation of the age-sex composition of a population. These pyramids consist of bars representing various age groups, usually at 5 years intervals. Length of the bar from central axis represents the percentage of population in an age group, showing males on the left of the axis and females on the right.

These pyramids enable us to know whether population of a place is dominated by younger or older people, males or females.

This helps the government in making future policies regarding old homes, educational institutions, medical facilities etc.

Logistic curve

One of the major reasons of studying demography is to be able to estimate the size of the population in future keeping in view the present size of the population. There are different methods to estimate the size of the population.

Logistic curve represents one of the mathematical methods to predict the size of the population at some future time. This curve fits best to populations that are growing under constraints of food and space but without any biological restriction on reproduction. It was first mentioned in 1838 by Verhulst while experimenting with the growth of insects under controlled experimental conditions and was given much later by Pearl and Reed in 1920.

Let P be the population at time t and P+ΔP be the population at time t+Δt. then the rate of growth in population at time t is defined as $dP/dt = \lim_{\Delta t \to 0} \Delta P/\Delta t$

Thus the relative growth rate can be expressed as $\frac{1}{P} \times \frac{dP}{dt}$

If the relative growth rate is constant i.e. $\frac{1}{P} \times \frac{dP}{dt} = c$ (21)

Then on solving the differential equation $\frac{d \ln P}{dt} = c$, we get

$\ln P = \int c\, dt = I_1 + ct$, where, I_1, is the constant of integration

=> $P = ke^{Ct}$, where k is a positive constant. (22)

On the other hand, if relative growth changes with time t then,

$\frac{1}{P} \times \frac{dP}{dt} = c(1\text{-}rP)$ (where, c and r, are positive constants) (23)

⇨ $\left(\frac{1}{P} + \frac{r}{1-rP}\right) \times \frac{dP}{dt} = c$ (24)

⇨ $\ln P - \ln(1\text{-}rP) = ct + I_2$, where, I_2, is the constant of integration

⇨ $\frac{P}{1-rP} = ke^{ct}$, where, k, is a positive constant.

⇨ $P = \dfrac{1}{r + \dfrac{1}{k} e^{-ct}}$ (25)

It can be easily seen from expression (25) that as $t \to \infty,\ p \to 1/r$ and as $t \to -\infty,\ p \to 0$.
Let U denote the upper limit to the population size then,

$$P = \frac{U}{1 + \frac{U}{k} e^{-ct}} \ldots\ldots \quad (26)$$

Suppose at t = β, P = U/2 then we get

$$\frac{U}{2} = \frac{U}{1 + \frac{U}{k} e^{-c\beta}}$$

⇨ $k = Ue^{-c\beta}$ (27)

⇨ $P = \dfrac{U}{1 + e^{c(\beta - t)}}$ or $\dfrac{1}{P} = \dfrac{1 + e^{c(\beta - t)}}{U}$ (28)

which is the general form of the logistic curve.

Fitting of logistic curve

Fitting of logistic curve involves the estimation of the parameters L, r and β. Assume that population figures are given for N equidistant points of time for t = 0,1,….,N-1. Then for fitting the logistic curve we shall be discussing the following two method:

Rhodes Method

One of the simplest methods is the method of Rhodes.
Substituting t = i-1 and t = i in the logistic curve expression (28) we get

$$\frac{1}{P_{i-1}} = \frac{1+e^{c(\beta - i + 1)}}{U} \text{ and } \frac{1}{P_i} = \frac{1+e^{c(\beta - i)}}{U}$$

$$\Rightarrow \frac{1}{P_i} = \frac{1-e^{-c}}{U} + e^{-c}\frac{1}{P_{i-1}} \quad (29)$$

which can be expressed in the form

$y_i = a + bx_i$ (where, a and b, are constant) (30)

which is a linear function of y and x and can be estimated by estimating the constants a and b

where $y_i = \dfrac{1}{P_i},\ \ x_i = \dfrac{1}{P_{i-1}},\ \ a = \dfrac{1-e^{-c}}{U},\ \ b = e^{-c}$

Finally, the constants U and c of the logistic equation can be estimated from the estimates of a and b and β can be estimated from the relation:

$$\beta = \frac{1}{Nc}\sum_{i=0}^{N-1} \ln\left(\frac{U}{P_i} - 1\right) + \frac{N-1}{2} \quad (31)$$

Pearl and Reed Method

The other method for fitting the logistic curve is the method of Pearl and Reed.
Here we consider three equidistant points in terms of time such that the logistic curve passes through these points i.e. consider three points of the form (t,P_t) as (i,P_i), $(i+n, P_{i+n})$, $(i+2n, P_{i+2n})$

Shifting the origin of time to $(0,P_0)$, (n, P_n), and $(2n, P_{2n})$ and substituting it in the expression (28) we get

$$\begin{aligned} \frac{1}{p_0} &= \frac{1+e^{c\beta}}{U} \\ \frac{1}{p_n} &= \frac{1+e^{c(\beta-n)}}{U} \\ \frac{1}{p_{2n}} &= \frac{1+e^{c(\beta-2n)}}{U} \end{aligned} \qquad (32)$$

Now subtracting second expression from first and third from second expression of (32) we get

$$\begin{aligned} s_1 &= \frac{1}{P_0} - \frac{1}{P_n} = \frac{1}{U} e^{c\beta}(1-e^{-cn}) \\ s_2 &= \frac{1}{P_n} - \frac{1}{P_{2n}} = \frac{1}{U} e^{c(\beta-n)}(1-e^{-cn}) \end{aligned} \qquad (33)$$

on dividing we get:

$$e^{cn} = \frac{s_1}{s_2} \qquad (34)$$

Also

$$\begin{aligned} \frac{Us_1}{e^{c\beta}} &= 1 - \frac{s_2}{s_1} \\ \Rightarrow \quad \frac{1}{U} &= \frac{1}{P_0} - \frac{s_1^2}{s_1 - s_2} \end{aligned} \qquad (35)$$

Once the estimates of U and c are obtained from the expressions (37) and (36) respectively, the estimate of β can be obtained from the relation $\beta = \frac{1}{c} \ln\left(\frac{U}{P_0} - 1\right)$

The estimates of U, c and β so obtained by the above method give only approximate values. To improve upon these estimates Pearl and Reed suggested a method based on least square technique.

Graduation of mortility curves

In practice, most of the data on vital events are taken from census and registration records. Even for the age specific death rates m_x, such sources are used, which is found to have various irregularities. The pivotal column of the life tables i.e. q_x column is based on the values of m_x. Thus one needs to smooth out these irregularities in order to construct life tables and perform various statistical analysis. Now force of mortality at age x can be defined as:

$$\mu_x = \lim_{\Delta x \to 0} \frac{1}{l_x} \times \frac{-\Delta l_x}{\Delta x} = -\frac{1}{l_x} \times \frac{dl_x}{dx} \qquad (36)$$

where l_x is the number of persons of exact age x; $-\Delta l_x$ is the number of persons who died between age x and $x+\Delta x$.

Also, one know $m_x = \frac{d_x}{L_x}$

where d_x is the number of deaths between age x and x+1 and L_x is the number of persons between age x and x+1.

Now, $L_x = \int_0^1 l_{x+t} dt$ (37)

Differentiating both the side w.r.t. x, we get

$$\frac{dL_x}{dx} = \frac{d}{dx} \int_0^1 l_{x+t} dt$$

$$= \int_0^1 \frac{d}{dx}(l_{x+t})dt, \text{ (assuming the validity of differentiation under the integration sign)}$$

$$\frac{dL_x}{dx} = \int_0^1 \frac{d}{dt}(l_{x+t})dt \text{ (since } l_{x+t} \text{ is continuous in x and t both)}$$

$$\frac{d}{dx} L_x = -dx \qquad (38)$$

Hence $M_x = -\frac{1}{L_x} \times \frac{dL_x}{dx}$;

$$\approx -\frac{1}{l_{x+\frac{1}{2}}} \times \frac{d}{dx} l_{x+\frac{1}{2}}$$

$$\Rightarrow M_x \approx \mu_{x+\frac{1}{2}} \qquad (39)$$

thus we shall consider μ_x instead of M_x in our future discussions.

Makeham's graduation formula

One of the formula to define μ_x was developed by Makeham under the assumption that the cause of death is either accident or decrease in resistance against disease. If h(x) represents the resistance to disease which is inversely proportional to the force of mortality, then

$$\mu_x = a + \frac{b}{h(x)} \qquad (40)$$

where. a,b > 0; and h(x), is a decreasing function of x.

Further, under the assumption that in a short interval of time a person looses a constant proportion of this force of resistance to disease, he took $\frac{1}{h(x)} \times \frac{d}{dx} h(x) = -c,\ (c>0)$

$$\Rightarrow h(x) = ke^{-cx} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(41)$$

Thus $\mu_x = a + \frac{b}{ke^{-cx}} = a + dr^x$ (a, d and r are constants) (42)

Using the relationship between μ_x and l_x, we have

$$l_x = km^x n^{r^x} \qquad (43)$$

Thus using this relationship, the values of l_x may be graduated in a left table.

Fitting of Makeham's graduation formula

Since the expression for l_x contains four unknown constants we need four independent equations to determine them. Consider four equispaced values of x (x = 0, n, 2n and 3n) to define four equations in terms of logarithms of l_x as:

$$\left.\begin{aligned} \log l_0 &= \log f + \log h \\ \log l_n &= \log f + n \log g + c_1^{\,n} \log h \\ \log l_{2n} &= \log f + 2n \log g + c_1^{\,2n} \log h \\ \log l_{3n} &= \log f + 3n \log g + c_1^{\,3n} \log h \end{aligned}\right\} \qquad (44)$$

in terms of differences:

$$\left.\begin{aligned} \Delta\log l_0 &= \log f + (c_1^{\,n}-1)\log h \\ \Delta\log l_n &= \log f + n \log g + c_1^{\,n}(c_1^{\,n}-1) \log h \\ \Delta\log l_{2n} &= \log f + 2n \log g + c_1^{\,2n}(c_1^{\,n}-1) \log h \end{aligned}\right\} \qquad (45)$$

and

$$\left.\begin{aligned} &\Delta^2\log l_0 = (c_1^{\,n}-1)^2 \log h \\ &\Delta^2\log l_n = c_1^{\,n}(c^n-1)^2 \log h \\ &\Rightarrow \Delta^2\log l_n = \Delta^2\log l_0 = c_1^n \end{aligned}\right\} \qquad (46)$$

Substituting the estimate of c_1 in one of the equation (46), we get the estimate of h. Now substituting these estimates of c_1 and h in one of the equations in expression (45) we get an estimate of g. These three estimates together when substituted in one of the equations in (44) give the estimate of f.

NOTE: To get better estimates one may use entire set of data by working with partial sums of observations viz. $S_0 = \sum_{x=0}^{n-1} \log l_x$, $S_1 = \sum_{x=n}^{2n-1} \log l_x$, $S_2 = \sum_{x=2n}^{3n-1} \log l_x$, $S_3 = \sum_{x=3n}^{4n-1} \log l_x$ instead of just four values of l_x.

Gompertz's graduation formula

Gompertz also developed a formula for μ_x and l_x prior to Makeham. However, he took into consideration only the factor resistance to disease, overlooking the factor of accidents altogether.

Practice questions

Question 1

Compute the standardized death rates for the following data:

Age Group	**Standard Population**	**Death Rate Per 1000**	
(in years)	(in Lakhs)	Country X	Country Y
0-4	107.20	21.44	16.22
5-9	95.03	19.01	14.15
10-14	92.59	18.52	13.74
15-19	90.44	18.09	13.37
20-24	87.96	17.59	12.95
25-29	85.17	17.03	12.48
30-34	60.53	12.11	8.29
35-39	73.62	14.72	10.51
40-44	65.84	13.17	9.19
45-49	58.07	11.61	7.87
50-54	42.15	8.43	5.17
55-59	41.90	8.38	5.12
60-64	33.55	6.71	3.70
65-69	25.38	5.08	2.31
70-74	17.92	3.58	1.05
75-79	11.62	2.32	2.02
80-84	6.77	1.35	2.85
85++	4.28	6.85	2.17

Question 2.

The following table gives a part of the census data of males in India. Complete the life table.

X	0	1	2	3	4	5	6	7	8	9	10
l_x	100000	86500	82992	81950	81388	80918	80512	80181	79882	79609	79348

Question 3.

Compute the GFR, TFR, and GRR for the following data assuming that for every 100 girls 112 boys are born

Age of Mother	Number of Women ('000)	Age specific fertility rate (per 1000)
15 – 19	221	102.3
20 – 24	207	173.9
25 – 29	171	162.5
30 – 34	153	44.0
35 – 39	136	102.9
40 – 44	114	47.1
45 – 49	95	21.2

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